SECTION I:

INVESTING IN PROJECTS

2

STATE-OF-THE-ART PROJECT APPRAISAL TECHNIQUES

Introduction

Shareholders supply funds to a firm for a reason. That reason, generally, is to receive a return on their precious resources. The return is generated by management using the finance provided to invest in real assets. It is vital for the health of the firm and the economic welfare of the finance providers that management employ the best techniques available when analyzing which of all the possible investment opportunities will give the best return.

Someone (or a group) within the organization may have to take the bold decision as to whether:

- it is better to build a new factory or extend the old;

- it is wiser to use an empty piece of land for a multi-story car park or to invest a larger sum and build a shopping center;

- whether shareholders would be better off if the firm returned their money in the form of dividends because shareholders can obtain a better return elsewhere; or

- the firm should pursue its expansion plan and invest in that new chain of hotels, or that large car showroom, or the new football stand.

These sorts of decisions require not only brave people, but informed people; individuals of the required caliber need to be informed about a range of issues: for example, the market environment and level of demand for the proposed activity, the internal environment, culture and capabilities of the firm, the types and levels of cost elements in the proposed area of activity, and, of course, an understanding of the risk and uncertainty appertaining to the project.

Cadbury Schweppes presumably considered all these factors before making their multi-million pound investments:

CADBURY SCHWEPPES

The 2002 annual report for Cadbury Schweppes shows that the company spent hundreds of millions investing in the business. The report describes the following investments in tangible and intangible assets:

'Capital expenditure in 2002 was £279m … The Group continued to implement a major project to standardise business systems and processes (Project PROBE) using a SAP platform. … The Group also carried out specific projects to increase production capacity in Mott's, Schweppes Spain and Cadbury Trebor Bassett. All these projects were funded from internal sources …'

Total marketing expenditure in 2002 was £547m. The company also spent £53m on major restructuring activities including the integration of Orangina into Schweppes France (£13m), of La Casera into Schweppes Spain (£10m), and Hollywood into Cadbury France (£10m). The group also devoted £32 million to research and development.

EXHIBIT 2.1 Cadbury Schweppes

Source: Cadbury Schweppes Annual Report and Form 20-F 2002

Bravery, information, knowledge and a sense of proportion are all essential ingredients when undertaking the onerous task of investing other people's money, but there is another element which is also crucial: that is, the employment of an investment appraisal technique which leads to the 'correct' decision; a technique which takes into account the fundamental considerations.

This chapter examines two approaches to evaluating investments within the firm. Both emphasize the central importance of the concept of the time value of money and are thus described as discounted cash flow (DCF) techniques. Net present value (NPV) and internal rate of return (IRR) are in common usage in most large commercial organizations and are regarded as more complete than the traditional techniques of payback and accounting rate of return (e.g. return on capital employed – ROCE). The relative merits and demerits of these alternative methods are discussed in Chapter 3. This chapter concentrates on gaining an understanding of how net present value and internal rate of return are calculated, as well as their theoretical under-pinnings.

How do you know if an investment generates value for shareholders?

If we accept that the objective of investment within the firm is to create value for its owners then the purpose of allocating money to a particular division or project is to generate a cash inflow in the future, significantly greater than the amount invested. Put most simply, the project appraisal decision is one involving the comparison of the amount of cash put into an investment with the amount of cash returned. The key phrase and the tricky issue is 'significantly greater than'. For instance, would you, as part-owner of a firm, be content if that firm asked you

> The project appraisal decision is one involving the comparison of the amount of cash put into an investment with the amount of cash returned.

to swap £10,000 of your hard-earned money for some new shares so that the management team could invest it only to hand back to you, in five years, the £10,000 plus £1,000? Is this a significant return? Would you feel that your wealth had been enhanced if you were aware that by investing the £10,000 yourself, by, for instance, lending to the government, you could have received a 5 percent return per year? Or that you could obtain a return of 15 percent per annum by investing in other shares on the stock market? Naturally, you would feel let down by a management team that offered a return of less than 2 percent per year when you had alternative courses of action that would have produced much more.

This line of thought is leading us to a central concept in finance and, indeed, in business generally – the time value of money. Investors have alternative uses for their funds and they therefore have an opportunity cost if money is invested in a corporate project. The *investor's opportunity cost* is the sacrifice of the return available on the best forgone alternative.

> The *investor's opportunity cost* is the sacrifice of the return available on the best forgone alternative.

Investments must generate at least enough cash for all investors to obtain their required returns. If they produce less than the investor's opportunity cost then the wealth of shareholders will decline.

Figure 2.1 summarizes the process of good investment appraisal. The acheivement of value or wealth creation is determined not only by the future cash flows to be derived from a project but also by the timing of those cash flows and by making an allowance for the fact that time has value.

Time is money

When people undertake to set aside money for investment something has to be given up now. For instance, if someone buys shares in a firm or lends to a business there is a sacrifice of consumption. One of the incentives to save is the possibility of gaining a higher level of future consumption by sacrificing some present consumption. So, some compensation is required to induce people to make a consumption sacrifice. Compensation will be required for at least three things:

■ **Time** That is, individuals generally prefer to have £1.00 today than £1.00 in five years' time. To put this formally: the utility of £1.00 now is greater than £1.00 received five years hence. Individuals are predisposed towards *impatience to consume* – they need an appropriate reward to begin the saving process. The rate of exchange between certain future consumption and certain current consumption is the *pure rate of interest* – this occurs even in a world of no inflation and no risk. If you lived in such a world you might be willing to sacrifice £100 of consumption now if you were compensated with £102.30 to be received in one year. This would mean that your *pure rate of interest* is 2.3 percent.

■ **Inflation** The price of time (or the interest rate needed to compensate for time preference) exists even when there is no inflation, simply because people generally prefer consumption now to consumption later. If there is inflation then the providers of finance will have to be compensated for that loss in purchasing power as well as for time.

FIGURE 2.1

Investment appraisal: objective, inputs and process
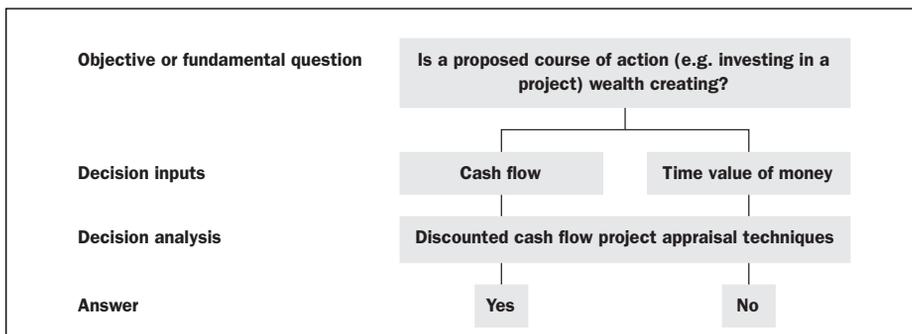
■ *Risk* The promise of the receipt of a sum of money some years hence gen-
erally carries with it an element of risk; the pay-out may not take place or
the amount may be less than expected. Risk simply means that the future
return has a variety of possible values. The issuer of a security, whether it be
a share, a bond or a bank account, must be prepared to compensate the
investor for time, inflation and risk involved, otherwise no one will be will-
ing to buy the security.

Take the case of Mrs Ann Investor who is considering a £1,000 one-year
investment and requires compensation for three elements of time value. First, a
return of 2.3 percent is required for the pure time value of money. Second, infla-
tion is anticipated to be 3 percent over the year. Thus, at time zero (t_0) £1,000
buys one basket of goods and services. To buy the same basket of goods and
services at time t_1 (one year later) £1,030 is needed. To compensate the investor
for impatience to consume and inflation the investment needs to generate a
return of 5.37 percent, that is:

$$(1 + 0.023)\,(1 + 0.03) - 1 = 0.0537$$

The figure of 5.37 percent may be regarded here as the risk-free return
(RFR), the interest rate that is sufficient to induce investment assuming no
uncertainty about the future cash flows.

Investors tend to view lending to reputable governments through the pur-
chase of bonds or bills as the nearest they are going to get to risk-free investing,
because these institutions have unlimited ability to raise income from taxes or to
create money. The RFR forms the bedrock for time value of money calculations
as the pure time value and the expected inflation rate affect all investments
equally. Whether the investment is in property, bonds, shares or a factory, if
expected inflation rises from 3 to 5 percent, then the investor's required return
on all investments will increase by 2 percent.

However, different investment categories carry different degrees of uncer-
tainty about the outcome of the investment. For instance, an investment on the
Russian stock market, with its high volatility, may be regarded as more risky
than the purchase of a share in AstraZeneca with its steady growth prospects.
Investors require different risk premiums on top of the RFR to reflect the per-
ceived level of extra risk. Thus:

Required return   =   RFR   +   Risk premium
(Time value of money)

In the case of Mrs Ann Investor, the risk premium pushes up the total return
required to, say, 9 percent giving full compensation for all three elements of the
time value of money.

Discounted cash flow

The net present value and internal rate of return techniques discussed in the rest of the chapter, both being discounted cash flow methods, take into account the time value of money. Table 2.1, which presents Project Alpha, suggests that on a straightforward analysis, Project Alpha generates more cash inflows than outflows. An outlay of £2,000 produces £2,400.

TABLE 2.1
Project Alpha, simple cash flow

Points in time (yearly intervals)	Cash flows (£)
0 Now	–2,000
1 (1 year from now)	+600
2	+600
3	+600
4	+600

However, we may be foolish to accept Project Alpha on the basis of this crude methodology. The £600 cash flows occur at different times, so are worth different amounts to a person standing at time zero. Quite naturally, such an individual would value the £600 received in one year more highly than the £600 received after four years. In other words, the present value of the pounds (at time zero) depends on when they are received.

It would be useful to convert all these different 'qualities' of pounds to a common currency, to some sort of common denominator. The conversion process is achieved by discounting all future cash flows by the time value of money, thereby expressing them as an equivalent amount received at time zero. The process of discounting relies on a variant of the compounding formula:

$$F = P(1+i)^n$$

where F = future value

P = present value

i = interest rate

n = number of years over which compounding takes place

Note: Please turn to Appendix 2.1 at this point to get to grips with the key mathematical tools that will be used in this chapter and throughout the rest of the book.

If a saver deposited £100 in a bank account paying interest at 8 percent per annum, after three years the account will contain £125.97:

$$F = 100 \, (1 + 0.08)^3 = £125.97$$

This formula can be changed so that we can answer the following question: 'How much must I deposit in the bank now to receive £125.97 in three years?'

$$P = \frac{F}{(1 + i)^n} \quad \text{or} \quad F \times \frac{1}{(1 + i)^n}$$

$$P = \frac{125.97}{(1 + 0.08)^3} = 100$$

In this second case we have *discounted* the £125.97 back to a present value of £100. If this technique is now applied to Project Alpha to convert all the money cash flows of future years into their present value equivalents the result is as follows (assuming that the time value of money is 19 percent).

TABLE 2.2
Project Alpha, discounted cash flow

Points in time (yearly intervals)	Cash flows (£)	Discounted cash flows (£)
0	–2,000	–2,000.00
1	+600	$\frac{600}{1 + 0.19} = +504.20$
2	+600	$\frac{600}{(1 + 0.19)^2} = +423.70$
3	+600	$\frac{600}{(1 + 0.19)^3} = +356.05$
4	+600	$\frac{600}{(1 + 0.19)^4} = +299.20$

When these future pounds are converted to a common denominator, this investment involves a larger outflow (£2,000) than inflow (£1,583.15). In other words the return on the £2,000 is less than 19 percent.

> **Technical aside**
>
> If your calculator has a 'powers' function (usually represented by x^y or y^x) then compounding and discounting can be accomplished relatively quickly. Alternatively, you may obtain discount factors from the table in Appendix II at the end of the book. If we take the discounting of the fourth year's cash flow for Alpha as an illustration:
>
> *Calculator*:
>
> $$\frac{1}{(1 + 0.19)^4} \times 600$$
>
> Input 1.19
>
> Press y^x (or x^y)
>
> Input 4
>
> Press =
>
> Display 2.0053
>
> Press $^1/_x$
>
> Display 0.4987
>
> Multiply by 600
>
> Answer 299.20.
>
> Using Appendix II, look down the column 19% and along the row 4 years to find discount factor of 0.4987:
>
> $0.4987 \times 600 = 299.20$

State-of-the-art technique 1: net present value

The conceptual justification for, and the mathematics of, the net present value method of project appraisal will be illustrated through an imaginary but realistic decision-making process at the firm of Hard Decisions plc. This example, in addition to describing the technique, demonstrates the centrality of some key concepts such as opportunity cost and time value of money and shows the wealth-destroying effect of ignoring these issues.

   Imagine you are the finance director of a large publicly quoted company called Hard Decisions plc. The board of directors agreed that the objective of the firm should be shareholder wealth maximization. Recently, the board appointed a new director, Mr Brightspark, as an 'ideas' man. He has a reputation as someone who can see opportunities where others see only problems. He has been hired especially to seek out new avenues for expansion and make better use of existing assets. In the past few weeks Mr Brightspark has been looking at some land that the company owns near the center of Birmingham. This is a ten-acre site on which the flagship factory of the firm once stood; but that was

30 years ago and the site is now derelict. Mr Brightspark announces to a board meeting that he has three alternative proposals concerning the ten-acre site.

Mr Brightspark stands up to speak: Proposal 1 is to spend £5m clearing the site, cleaning it up, and decontaminating it. [The factory that stood on the site was used for chemical production.] It would then be possible to sell the ten acres to property developers for a sum of £12m in one year's time. Thus, we will make a profit of £7m over a one-year period.

Proposal 1: Clean up and sell – Mr Brightspark's figures

Clearing the site plus decontamination payable, t_0	–£5m
Sell the site in one year, t_1	£12m
Profit	£7m

The chairman of the board stops Mr Brightspark at that point and turns to you, in your capacity as the financial expert on the board, to ask what you think of the first proposal. Because you have studied your *Handbook of Corporate Finance* assiduously you are able to make the following observations:

- **Point 1** This company is valued by the stock market at £100m because our investors are content that the rate of return they receive from us is consistent with the going rate for our risk class of shares; that is, 15 percent per annum. In other words, the opportunity cost for our shareholders of buying shares in this firm is 15 percent. (Hard Decisions is an all-equity firm, no debt capital has been raised.) The alternative to investing their money with us is to invest it in another firm with similar risk characteristics yielding 15 percent per annum. Thus, we may take this *opportunity cost of capital* as our minimum required return from any project we undertake. This idea of opportunity cost can perhaps be better explained by the use of a diagram (*see* Figure 2.2).

FIGURE 2.2
The investment decision: alternative uses of firm's funds
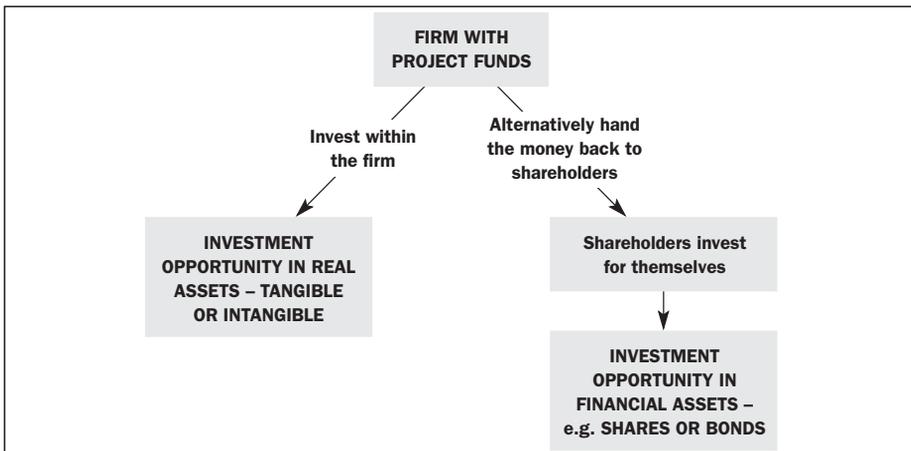
If we give a return of less than 15 percent then shareholders will lose out because they can obtain 15 percent elsewhere, so will suffer an opportunity cost.

We, as managers of shareholders' money, need to use a discount rate of 15 percent for any project of the same risk class that we analyze. The discount rate is the opportunity cost of investing in the project rather than the capital markets, for example, buying shares in other firms giving a 15 percent return. Instead of accepting this project the firm can always give the cash to the shareholders and let them invest it in financial assets.

■ ***Point 2*** I believe I am right in saying that we have received numerous offers for the ten-acre site over the past year. A reasonable estimate of its immediate sale value would be £6m. That is, I could call up one of the firms keen to get its hands on the site and squeeze out a price of about £6m. This £6m is an opportunity cost of the project, in that it is the value of the best alternative course of action. Thus, we should add to Mr Brightspark's £5m of clean-up costs, the £6m of opportunity cost because we are truly sacrificing £11m to put this proposal into operation. If we did not go ahead with Mr Brightspark's plan, but sold the site as it is, we could raise our bank balance by £6m, plus the £5m saved by not paying clean-up costs.

Proposal 1: Clean up and sell – Time t_0 cash flows

Immediate sale value (opportunity cost)	£6m
Clean up, etc.	£5m
Total sacrifice at t_0	£11m

■ ***Point 3*** I can accept Mr Brightspark's final sale price of £12m as being valid in the sense that he has, I know, employed some high quality experts to do the sum, but I do have a problem with comparing the initial outlay *directly* with the final cash flow on a simple *nominal* sum basis. The £12m is to be received in one year's time, whereas the £5m is to be handed over to the clean-up firm immediately, and the £6m opportunity cost sacrifice, by not selling the site, is being made immediately.

If we were to take the £11m initial cost of the project and invest it in financial assets of the same risk class as this firm, giving a return of 15 percent, then the value of that investment at the end of one year would be £12.65m. The calculation for this:

$$F = P\,(1 + k)$$

where k = the opportunity cost of capital:

$$11\,(1 + 0.15) = £12.65m$$

This is more than the return promised by Mr Brightspark.

Another way of looking at this problem is to calculate the net present value of the project. We start with the classic formula for net present value:

$$NPV = F_0 + \frac{F_1}{(1 + k)^n}$$

where F_0 = cash flow at time zero (t_0), and

F_1 = cash flow at time one (t_1), one year after time zero

n = number of years from the present for that cash flow – in this case, one:

$$NPV = -11 + \frac{12}{1 + 0.15} = -11 + 10.43 = -0.56m$$

All cash flows are expressed in the common currency of pounds at time zero. Everything is in present value terms. When the positives and negatives are netted out we have the *net* present value. The decision rules for net present value are:

NPV ≥ 0	Accept
NPV < 0	Reject

An investment proposal's net present value is derived by discounting the future net cash receipts at a rate which reflects the value of the alternative use of the funds, summing them over the life of the proposal and deducting the initial outlay.

In conclusion, Ladies and Gentlemen, given the choice between:

(a) selling the site immediately raising £6m and saving £5m of expenditure – a total of £11m, or

(b) developing the site along the lines of Mr Brightspark's proposal,

I would choose to sell it immediately because £11m would get a better return elsewhere.

The chairman thanks you and asks Mr Brightspark to explain Project Proposal 2. Proposal 2 consists of paying £5m immediately for a clean-up. Then, over the next two years, spending another £14m building an office complex. Tenants would not be found immediately on completion of the building. The office units would be let gradually over the following three years. Finally, when the office complex is fully let, in six years' time, it would be sold to an institution, such as a pension fund, for the sum of £40m (*see* Table 2.3).

Mr Brightspark claims an almost doubling of the money invested (£25m invested over the first two years leads to an inflow of £47m). The chairman turns to you and asks: Is this project really so beneficial to our shareholders?

TABLE 2.3

Proposal 2: Office complex – Mr Brightspark's figures

Points in time (yearly intervals)	Cash flows (£m)	Event
0 (now)	−5	Clean-up costs
0 (now)	−6	Opportunity cost
1	−4	Building cost
2	−10	Building cost
3	+1	Net rental income, $\frac{1}{4}$ of offices let
4	+2	Net rental income, $\frac{1}{2}$ of offices let
5	+4	Net rental income, all offices let
6	+40	Office complex sold
TOTAL	+22	Inflow £47m
		Outflow £25m
PROFIT	22	

(*Note*: Mr Brightspark has accepted the validity of your argument about the opportunity cost of the alternative 'project' of selling the land immediately and has quickly added this –£6m to the figures.)

You reply: The message rammed home to me by my finance book was that the best method of assessing whether a project is shareholder wealth enhancing is to discount all its cash flows at the opportunity cost of capital. This will enable a calculation of the net present value of those cash flows.

$$\text{NPV} = F_0 + \frac{F_1}{1 + k} + \frac{F_2}{(1 + k)^2} + \frac{F_3}{(1 + k)^3} \cdots + \frac{F_n}{(1 + k)^n}$$

So, given that Mr Brightspark's figures are true cash flows, I can calculate the NPV of Proposal 2 – *see* Table 2.4.

Because the NPV is less than 0, we would serve our shareholders better by selling the site and saving the money spent on clearing and building and putting that money into financial assets yielding 15 percent per annum. Shareholders would end up with more in Year 6.

The chairman thanks you and asks Mr Brightspark for his third proposal. Proposal 3 involves the use of the site for a factory to manufacture the product 'Worldbeater'. Mr Brightspark says we have been producing 'Worldbeater' from our Liverpool factory for the past ten years. Despite its name, we have confined the selling of it to the UK market. I propose the setting up of a second 'Worldbeater' factory which will serve the European market. The figures are as follows (*see* Table 2.5).

TABLE 2.4

Proposal 2: Net present values

Points in time (yearly intervals)	Cash flows (£m)		Discounted cash flows (£m)
0	−5		−5
0	−6		−6
1	−4	$\dfrac{-4}{(1 + 0.15)}$	−3.48
2	−10	$\dfrac{-10}{(1 + 0.15)^2}$	−7.56
3	1	$\dfrac{1}{(1 + 0.15)^3}$	0.66
4	2	$\dfrac{2}{(1 + 0.15)^4}$	1.14
5	4	$\dfrac{4}{(1 + 0.15)^5}$	1.99
6	40	$\dfrac{40}{(1 + 0.15)^6}$	17.29
Net present value			−0.96

TABLE 2.5

Manufacture of 'Worldbeater' – Mr Brightspark's figures

Points in time (yearly intervals)	Cash flows (£m)	Event
0	−5	Clean-up
0	−6	Opportunity cost
1	−10	Factory building
2	0	Net cash flows from operating
3 to infinity	+5	Net cash flows from additional sales of 'Worldbeater'

Note: Revenue is gained in Year 2 from sales but this is exactly offset by the cash flows created by the costs of production and distribution. The figures for Year 3 and all subsequent years are net cash flows, that is, cash outflows are subtracted from cash inflows generated by sales.

The chairman turns to you and asks your advice.

You reply: Worldbeater is a well-established product and has been very successful. I am happy to take the cash flow figures given by Mr Brightspark as the basis for my calculations, which are as follows (*see* Table 2.6).

TABLE 2.6
Worldbeater Net Present Value

Points in time (yearly intervals)	Cash flows (£m)		Discounted cash flows (£m)
0	−11		−11
1	−10	$\dfrac{-10}{(1 + 0.15)}$	−8.7
2	0		
3 to infinity	5	Value of perpetuity at time t_2:	
		$P = \dfrac{F}{k} = \dfrac{5}{0.15} = 33.33.$	
		This 33.33 has to be discounted back two years:	
		$\dfrac{33.33}{(1 + 0.15)^2}$	= 25.20
Net present value			+ 5.5

Note: The perpetuity formula can be used on the assumption that the first payment arises one year from the time at which we are valuing. So, if the first inflow arises at time 3 we are valuing the perpetuity as though we are standing at time 2. The objective of this exercise is not to convert all cash flows to time 2 values, but rather to time 0 value. Therefore, it is necessary to discount the perpetuity value by two years. (If these calculations are confusing you are advised to read the mathematical Appendix 2.1 at the end of this chapter.)

This project gives an NPV that is positive, so is shareholder wealth enhancing because it gives a rate of return that is greater than 15 percent per annum. It provides a return of 15 percent plus a present value of £5.5m. Based on these figures I would recommend that the board examine Proposal 3 in more detail.

The chairman thanks you and suggests that this proposal be put to the vote.

Mr Brightspark (interrupts): Just a minute, are we not taking a lot on trust here? Our finance expert has stated that the way to evaluate these proposals is by using the NPV method, but in the firms where I have worked in the past, the internal rate of return (IRR) method of investment appraisal was used. I would like to see how these three proposals shape up when the IRR calculations are done.

The chairman turns to you and asks you to explain the IRR method, and to apply it to the figures provided by Mr Brightspark ….

Before continuing this boardroom drama it might be useful at this point to broaden the understanding of NPV by considering two worked examples.

Worked example 2.1
CAMRAT PLC

Camrat plc requires a return on investment of at least 10% per annum over the life of a project to meet the opportunity cost of its shareholders (Camrat is financed entirely by equity). The dynamic and thrusting strategic development team have been examining the possibility of entering the new market area of mosaic floor tiles. This will require an immediate outlay of £1m for factory purchase and tooling-up which will be followed by *net* (i.e. after all cash outflows (wages, variable costs, etc.)) cash inflows of £0.2m in one year, and £0.3m in two years' time. Thereafter, annual net cash inflows will be £180,000.

Required

Given these cash flows, will this investment provide a 10% return (per annum) over the life of the project? Assume for simplicity that all cash flows arise on anniversary dates.

Answer

First, lay out the cash flows with precise timing. (Note: the assumption that all cash flows arise on anniversary dates allows us to do this very simply.)

Points in time (yearly intervals)	0	1	2	3 to infinity
Cash flows (£)	−1m	0.2m	0.3m	0.18m

Second, discount these cash flows to their present value equivalents.

Points in time	0	1	2	3 to infinity
	F_0	$\dfrac{F_1}{1+k}$	$\dfrac{F_2}{(1+k)^n}$	$\dfrac{F_3}{k} \times \dfrac{1}{(1+k)^2}$
	−1m	$\dfrac{0.2}{1+0.1}$	$\dfrac{0.3}{(1+0.1)^2}$	$\dfrac{0.18}{0.1}$
				This discounts back two years:
				$\dfrac{0.18/0.1}{(1+0.1)^2}$
	−1m	0.1818	0.2479	$\dfrac{1.8}{(1.1)^2} = 1.4876$

Third, net out the discounted cash flows to give the net present value.

$$
\begin{array}{r}
-1.0000 \\
+0.1818 \\
+0.2479 \\
\underline{+1.4876} \\
\end{array}
$$

Net present value $\underline{+0.9173}$

Conclusion

The positive NPV result demonstrates that this project gives not only a return of 10% per annum but a large surplus above and beyond a 10% per annum return. This is an extremely attractive project: on a £1m investment the surplus generated beyond the opportunity cost of the shareholders (their time value of money) is £917,300; by accepting this project we would increase shareholder wealth by this amount.

Worked example 2.2
ACTARM PLC

Actarm plc is examining two projects, A and B. The cash flows are as follows:

	A £	B £
Initial outflow, t_0	240,000	240,000
Cash inflows:		
Time 1 (one year after t_0)	200,000	20,000
Time 2	100,000	120,000
Time 3	20,000	220,000

Using discount rates of 8%, and then 16%, calculate the NPVs and state which project is superior. Why do you get a different preference depending on the discount rate used?

Answer

Using 8% as the discount rate:

$$
\text{NPV} = F_0 + \frac{F_1}{1+k} + \frac{F_2}{(1+k)^2} + \frac{F_3}{(1+k)^3}
$$

Project A

$$
-240,000 + \frac{200,000}{1+0.08} + \frac{100,000}{(1+0.08)^2} + \frac{20,000}{(1+0.08)^3}
$$

$$
\mathbf{-240,000 + 185,185 + 85,734 + 15,877 = +\,£46,796}
$$

Project B

$$-240,000 + \frac{20,000}{1 + 0.08} + \frac{120,000}{(1 + 0.08)^2} + \frac{220,000}{(1 + 0.08)^3}$$

–240,000 + 18,519 + 102,881 + 174,643   = +£56,043

Using an 8% discount rate, both projects produce positive NPVs so would enhance shareholder wealth. However, Project B is superior because it creates more value than Project A. If accepting one project excludes the possibility of accepting the other, then B is preferred.

Using 16% as the discount rate:

Project A

$$-240,000 + \frac{200,000}{1.16} + \frac{100,000}{(1.16)^2} + \frac{20,000}{(1.16)^3}$$

–240,000 + 172,414 + 74,316 + 12,813   = +£19,543

Project B

$$-240,000 + \frac{20,000}{1.16} + \frac{120,000}{(1.16)^2} + \frac{220,000}{(1.16)^3}$$

–240,000 + 17,241 + 89,180   + 140,945   = +£7,366

With a 16% discount rate Project A generates more shareholder value and so would be preferred to Project B. This is despite the fact that Project B, in pure undiscounted cash flow terms, produces an additional £40,000.

The different ranking (order of superiority) occurs because Project B has the bulk of its cash flows occurring towards the end of the project's life. These large distant cash flows, when discounted at a high discount rate, become relatively small compared with those of Project A, which has its high cash flows discounted by only one year. Obtaining the appropriate discount rate to use in calculations of this kind is discussed in Chapter 10.

State-of-the-art technique 2: internal rate of return

We now return to Hard Decisions plc. The chairman has asked you to explain internal rate of return (IRR).

You respond: The internal rate of return is a very popular method of project appraisal and it has much to commend it. In particular it takes into account the time value of money. I am not surprised to find that Mr Brightspark has encountered this appraisal technique in his previous employment. Basically, what the IRR tells you is the rate of interest you will receive by putting your money into a project. It describes by how much the cash inflows exceed the cash outflows on an annualized percentage basis, taking account of the timing of those cash flows.

The internal rate of return is the rate of return, r, that equates the present value of future cash flows with the outlay (or, for some projects, it equates discounted future cash outflows with initial inflow):

Outlay = Future cash flows discounted at rate r

Thus:

$$F_0 = \frac{F_1}{1+r} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3} \cdots \frac{F_n}{(1+r)^n}$$

IRR is also referred to as the 'yield' of a project.

Alternatively, the internal rate of return, r, is the discount rate at which the net present value is zero. It is the value for r that makes the following equation hold:

$$F_0 + \frac{F_1}{1+r} + \frac{F_2}{(1+r)^2} + \frac{F_3}{(1+r)^3} \cdots \frac{F_n}{(1+r)^n} = 0$$

(*Note*: in the first formula F_0 is expressed as a positive number, whereas in the second it is usually a negative.)

These two equations amount to the same thing. They both require knowledge of the cash flows and their precise timing. The element that is unknown is the rate of interest that will make the time-adjusted outflows and inflows equal to each other.

I apologize, ladies and gentlemen, if this all sounds like too much jargon. Perhaps it would be helpful if you could see the IRR calculation in action. Let's apply the formula to Mr Brightspark's Proposal 1.

Proposal 1: Internal rate of return

Using the second version of the formula, our objective is to find an r that makes the discounted inflow at time 1 of £12m plus the initial £11m outflow equal to zero:

$$F_0 + \frac{F_1}{1+r} = 0$$

$$-11 + \frac{12}{1+r} = 0$$

The method I would recommend for establishing r is trial and error (assuming we do not have the relevant computer program available). So, to start with, simply pick an interest rate and plug it into the formula. Let's try 5 percent:

$$-11 + \frac{12}{1+0.05} = £0.42857\text{m or }£428,571$$

A 5 percent rate is not correct because the discounted cash flows do not total to zero. The surplus of approximately £0.43m suggests that a higher interest rate will be more suitable. This will reduce the present value of the future cash inflow. Let's try 10 percent:

$$-11 + \frac{12}{1 + 0.1} = -£0.0909 \text{ or } -£90,909$$

Again, we have not hit on the correct discount rate. Let's try 9 percent:

$$-11 + \frac{12}{1 + 0.09} = +£0.009174 \text{ or } +£9,174$$

The last two calculations tell us that the interest rate that equates to the present value of the cash flows lies somewhere between 9 and 10 percent. The precise rate can be found through interpolation.

First, display all the facts so far established:

Discount rate, r	9%	?	10%
Net present value	+£9,174	0	–£90,909
Point	A	B	C

There is a yield rate (r) that lies between 9 and 10 percent that will produce an NPV of zero. The way to find that interest rate is to first find the distance between points A and B, as a proportion of the entire distance between points A and C.

$$\frac{A \to B}{A \to C} = \frac{9,174 - 0}{9,174 + 90,909} = 0.0917$$

The ? lies at a distance of 0.0917 away from the 9 percent point.

IRR:

$$= 9 + \left(\frac{9,174}{100,083} \right) \times (10 - 9) = 9.0917 \text{ percent}$$

To double-check our result:

$$-11 + \frac{12}{1 + 0.090917}$$

$$-11 + 11 = 0$$

The IRR decision rules

The rule for internal rate of return decisions is:

■ **If $k > r$ reject** If the opportunity cost of capital (k) is greater than the internal rate of return (r) on a project then the investor is better served by not going ahead with the project and applying the money to the best alternative use.

■ **If $k \leq r$ accept** Here, the project under consideration produces the same or a higher yield than investment elsewhere for a similar risk level.

The IRR of Proposal 1 is 9.091 percent, which is below the 15 percent opportunity cost of capital used by Hard Decisions plc. Using the IRR method as well as the NPV method, this project should be rejected.

It might be enlightening to consider the relationship between NPV and IRR. Table 2.7 shows what happens to NPV as the discount rate is varied between zero and 10 percent for Proposal 1. At a zero discount rate the £12m received in one year is not discounted at all, so the NPV of £1m is simply the difference between the two cash flows. When the discount rate is raised to 10 percent the present value of the Year 1 cash flow becomes less than the current outlay. Where the initial outflow equals the *discounted* future inflows, i.e. when NPV is zero, we can read off the internal rate of return.

TABLE 2.7

The relationship between NPV and the discount rate (using Proposal 1's figures)

Discount rate (%)	NPV
10	–90,909
9.0917	0
9	9,174
8	111,111
7	214,953
6	320,755
5	428,571
4	538,461
3	650,485
2	764,706
1	881,188
0	1,000,000

Proposal 2: IRR

To calculate the IRR for Proposal 2 we first lay out the cash flows in the discount formula:

$$-11 + \frac{-4}{(1+r)} + \frac{-10}{(1+r)^2} + \frac{1}{(1+r)^3} + \frac{2}{(1+r)^4} + \frac{4}{(1+r)^5} + \frac{40}{(1+r)^6} = 0$$

Then we try alternative discount rates to find a rate, r, that gives a zero NPV:

Try 14 per cent:

NPV (approx.) = –£0.043 or –£43,000

At 13 per cent:

NPV = £932,000

Interpolation is required to find an internal rate of return accurate to at least one decimal place:

Discount rate

r	13%	?	14%
NPV	+932,000	0	–43,000

$$13 + \frac{932,000}{975,000} \times (14 - 13) = 13.96\%$$

This project produces an IRR (13.96%), which is less than the opportunity cost of shareholders' funds (15%); so it should be rejected under the IRR method. Because the line in Figure 2.3 is curved, it is important to have only a fairly small gap in trial and error interest rates prior to interpolation. The interpolation formula assumes a straight line between the two discount rates chosen. The effect of taking a wide range of interest rates can be illustrated if we calculate on the basis of 5 and 30 percent.

At 5%, NPV of Project 2 = £11.6121m.

At 30%, NPV of Project 2 = –£9.4743m.

FIGURE 2.3
Graph of NPV for Proposal 2
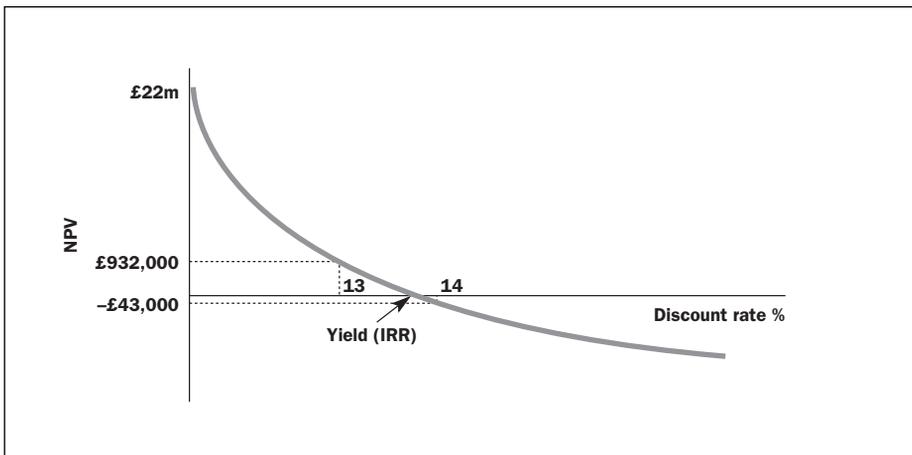
Linear interpolation

Discount rate

r	5%	?	30%
NPV	+11.6121	0	−9.4743

$$5 + \left(\frac{11.6121}{11.6121 + 9.4743} \right)(30 - 5) = 18.77\%$$

The non-linearity of the relationship between NPV and the discount rate has created an IRR almost 5 percent removed from the true IRR – *see* Figure 2.4. This could lead to an erroneous acceptance of this project given the company's hurdle rate of 15 percent. In reality this project yields less than the company could earn by placing its money elsewhere for the same risk level.

Proposal 3: IRR

$$F_0 + \frac{F_1}{1 + r} + \frac{F_3 / r}{(1 + r)^2} = 0$$

Try 19 percent:

$$-11 + \frac{-10}{1 + 0.19} + \frac{5 / 0.19}{(1 + 0.19)^2} = -£0.82m$$

Try 18 percent:

$$-11 + \frac{-10}{1 + 0.18} + \frac{5 / 0.18}{(1 + 0.18)^2} = +£0.475m$$

FIGURE 2.4

The accuracy of the IRR may depend on the size of the gap between the discount rates used in the interpolation calculation
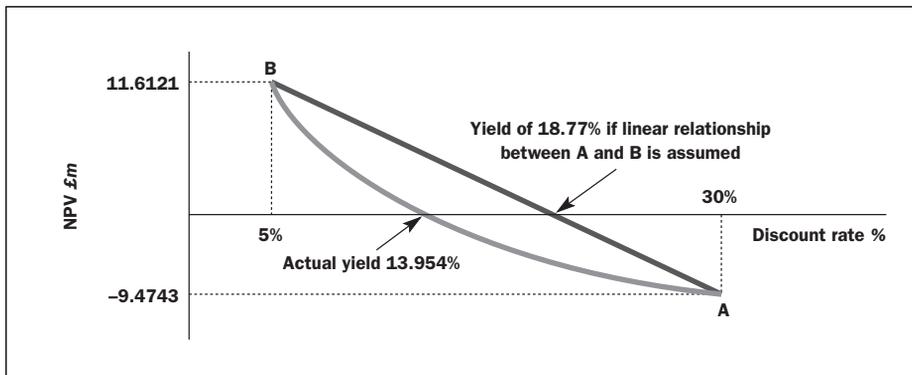
Linear Interpolation:

r	18%	?	19%
NPV	+475,000	0	–820,000

$$18 + \frac{475,000}{1,295,000} \times (19 - 18) = 18.37\%$$

Project 3 produces an internal rate of return of 18.37 percent which is higher than the opportunity cost of capital and therefore is to be commended.

We temporarily leave the saga of Mr Brightspark and his proposals to reinforce understanding of NPV and IRR through the worked example of Martac plc.

Worked example 2.3
MARTAC PLC

Martac plc is a manufacturer of *Martac-aphro*. Two new automated process machines used in the production of Martac have been introduced to the market, the CAM and the ATR. Both will give cost savings over existing processes:

	CAM £000s	ATR £000s
Initial cost (machine purchase and installation, etc.)	120	250
Cash flow savings:		
At Time 1 (one year after the initial cash outflow)	48	90
At Time 2	48	90
At Time 3	48	90
At Time 4	48	90

All other factors remain constant and the firm has access to large amounts of capital. The required return on projects is 8%.

Required:
(a) Calculate the IRR for CAM.
(b) Calculate the IRR for ATR.
(c) Based on IRR which machine would you purchase?
(d) Calculate the NPV for each machine.
(e) Based on NPV which machine would you buy?
(f) Is IRR or NPV the better decision tool?

Answers

In this problem the total cash flows associated with the alternative projects are not given. Instead the incremental cash flows are provided, for example, the additional savings available over the existing costs of production. This, however, is sufficient for a decision to be made about which machine to purchase.

(a) IRR for CAM

$$F_0 + \frac{F_1}{1 + r} + \frac{F_2}{(1 + r)^2} + \frac{F_3}{(1 + r)^3} + \frac{F_4}{(1 + r)^4} = 0$$

Try 22%:

$$-120,000 + 48,000 \times \text{annuity factor (af) for 4 years @ 22\%}$$

(*See* Appendix 2.1 (p. 55) for annuity calculations and Appendix III (p. 631) for an annuity table.)

The annuity factor tells us the present value of four lots of £1 received at four annual intervals. This is 2.4936, meaning that the £4 in present value terms is worth just over £2.49.

$$-120,000 + 48,000 \times 2.4936 = -£307.20$$

Try 21%:

$$-120,000 + 48,000 \times \text{annuity factor (af) for 4 years @ 21\%}$$

$$-120,000 + 48,000 \times 2.5404 = +£1,939.20$$

Interpolation

Discount rate

$$21 + \left(\frac{1939.2}{1939.2 + 307}\right)(22 - 21) = 21.86\%$$

(b) IRR for ATR

Try 16%:

$$-250,000 + 90,000 \times 2.7982 = +£1,838$$

Try 17%:

$$-250,000 + 90,000 \times 2.7432 = -£3,112$$

Discount rate, r

<pre>
         16%   ?   17%
     ┼─────────────────┼──────────────┼
NPV   +1,838   0   −3,112
</pre>

$$16 + \left(\frac{1{,}838}{1{,}838 + 3{,}112} \right) \times (17-16) = 16.37\%$$

(c) Choice of machine on basis of IRR

If IRR is the only decision tool available then as long as the IRRs exceed the discount rate (or cost of capital) the project with the higher IRR might appear to be the preferred choice. In this case CAM ranks higher than ATR.

(d) NPV for machines: CAM

$$-120{,}000 + 48{,}000 \times 3.3121 = +\pounds 38{,}981$$

NPV for ATR

$$-250{,}000 + 90{,}000 \times 3.3121 = +\pounds 48{,}089$$

(e) Choice of machine on basis of NPV

ATR generates a return which has a present value of £48,089 in addition to the minimum return on capital required. This is larger than for CAM and therefore ATR ranks higher than CAM if NPV is used as the decision tool.

(f) Choice of decision tool

This problem has produced conflicting decision outcomes, which depend on the project appraisal method employed. NPV is the better decision-making technique because it measures in absolute amounts of money. That is, it gives the increase in shareholder wealth available by accepting a project. In contrast, IRR expresses its return as a percentage which may result in an inferior low-scale project being preferred to a higher-scale project. So, if you cannot undertake both projects the one that returns most to shareholders is the one with the highest NPV rather than the highest IRR

Choosing between NPV and IRR

We now return to Hard Decisions plc.

Mr Brightspark: I have noticed your tendency to prefer NPV to any other method. Yet, in the three projects we have been discussing, NPV and IRR give the same decision recommendation. That is, reject Projects 1 and 2, and accept Project 3. So, why not use IRR more often?

You reply: It is true that the NPV and IRR methods of capital investment appraisal are closely related. Both are 'time-adjusted' measures of profitability. The NPV and IRR methods gave the same result in the cases we have considered today because the problems associated with the IRR method are not present in the figures we have been working with. In the appraisal of other projects we may encounter the severe limitations of the IRR method and therefore I prefer to stick to the theoretically superior NPV technique.

I will illustrate two of the most important problems, multiple solutions and ranking.

Multiple solutions

There may be a number of possible IRRs. This can be explained by examining the problems Mr Flummoxed is having (*see* Worked example 2.4).

Worked example 2.4
MR FLUMMOXED

Mr Flummoxed of Deadhead plc has always used the IRR method of project appraisal. He has started to have doubts about its usefulness after examining the proposal for 'Project Oscillation'.

Project oscillation

Points in time (yearly intervals)	0	1	2
Cash flow	–3,000	+15,000	–13,000

Internal rates of return are found at 11.56% and 288.4%.

Given that Deadhead plc has a required rate of return of 20%, it is impossible to decide whether to implement Project Oscillation using an unadjusted IRR methodology.

The cause of multiple solutions is unconventional cash flows. Conventional cash flows occur when an outflow is followed by a series of inflows or a cash inflow is followed by a series of cash outflows. Unconventional cash flows are a series of cash flows with more than one change in sign. In the case of Project Oscillation the sign changes from negative to positive once, and from positive to negative once. These two sign changes provide a clue to the number of possible solutions or IRRs. Multiple yields can be adjusted for while still using the IRR method, but the simplest approach is to switch to the NPV method.

Ranking

The IRR decision rule does not always rank projects in the same way as NPV. Sometimes it is important to find out, not only which project gives a positive return, but which one gives the greater positive return. For instance, projects may be mutually exclusive, that is, only one may be undertaken and a choice has to be made. The use of IRR alone sometimes leads to a poor choice (*see* Table 2.8).

TABLE 2.8
Illustration of the IRR ranking problem

Project	Cash flows £m		IRR%	NPV (at 15%)
	Time 0	*One year later*		
A	−20	+40	100%	+14.78m
B	−40	+70	75%	+20.87m

NPV at different discount rates

Discount rate (%)	Project A	Project B
0	20	30
20	13.33	18.33
50	6.67	6.67
75	2.86	0
100	0	−5
125	−2.22	−8.89

It is clear that the ranking of the projects by their IRRs is constant at 75 and 100 percent, regardless of the opportunity cost of capital (discount rate). Project A is always better. On the other hand, ranking the projects by the NPV method is not fixed. The NPV ranking depends on the discount rate. If the discount rate used in the NPV calculation is higher than 50 percent, the ranking under both IRR and NPV would be the same, i.e. Project A is superior. If the discount rate falls below 50 percent, Project B is the better choice. One of the major elements leading to the theoretical dominance of NPV is that it takes into account the scale of investment; the shareholders are made better off by £20.87m by undertaking Project B because the initial size of the project is larger. NPVs are measured in absolute amounts.

Conclusion

This chapter has provided insight into the key factors to consider when an organization is contemplating using financial (or other) resources for investment. The

analysis has been based on the assumption that the objective of any such invest-ment is to maximize economic benefits to the owners of the enterprise. To achieve such an objective requires allowance for the opportunity cost of capital or time value of money as well as robust analysis of relevant cash flows. Given that time has a value, the precise timing of cash flows is important for project analysis. The net present value (NPV) and internal rate of return (IRR) methods of project appraisal are both discounted cash flow techniques so allow for the time value of money. However, the IRR method does present problems in a few special circumstances and so the theoretically preferred method is NPV.

NPV requires diligent studying and thought to be fully understood, and there-fore it is not surprising to find in the workplace a bias in favour of communicating a project's viability in terms of percentages. In fact, most large organizations use three or four methods of project appraisal, rather than rely on only one for both rigorous analysis and communication – *see* Chapter 3 for more detail. The funda-mental conclusion of this chapter is that the best method to maximize shareholder wealth in assessing investment projects is net present value.

APPENDIX 2.1

Mathematical tools for finance

The purpose of this Appendix is to explain essential mathematical skills that are needed for the rest of the book. The author has no love of mathematics for its own sake so only those techniques of direct relevance to the subject matter of this textbook are covered in this section.

Simple and compound interest

When there are time delays between receipt and payment of financial sums we need to make use of the concepts of simple and compound interest.

Simple interest

Interest is paid only on the original principal. No interest is paid on the accumu-lated interest payments.

Example 1

Suppose that a sum of £10 is deposited in a bank account that pays 12 percent per annum. At the end of year 1 the investor has £11.20 in the account. That is:

$$F = P(1 + i)$$

$$11.20 = 10(1 + 0.12)$$

where F = Future value, P = Present value, i = Interest rate.
The initial sum, called the principal, is multiplied by the interest rate to give the annual return.
At the end of five years:

$$F = P(1 + in)$$

where n = number of years. Thus:

$$16 = 10(1 + 0.12 \times 5)$$

Note from the example that the 12 percent return is a constant amount each year. Interest is not earned on the interest already accumulated from previous years.

Compound interest

The more usual situation in the real world is for interest to be paid on the sum that accumulates – whether or not that sum comes from the principal or from the interest received in previous periods. Interest is paid on the accumulated interest and principal.

Example 2

An investment of £10 is made at an interest rate of 12 percent with the interest being compounded. In one year the capital will grow by 12 percent to £11.20. In the second year the capital will grow by 12 percent, but this time the growth will be on the accumulated value of £11.20 so will amount to an extra £1.34. At the end of two years:

$$F = P(1 + i)(1 + i)$$
$$F = 11.20(1 + i)$$
$$F = 12.54$$

Alternatively,

$$F = P(1 + i)^2$$

Table 2.9 displays the future value of £1 invested at a number of different interest rates and for alternative numbers of years. (This is an extract from Appendix I at the end of the book.)

From the second row of the table we can read that £1 invested for two years at 12 percent amounts to £1.2544. The investment of £10 provides a future capital sum 1.2544 times the original amount:

$$£10 \times 1.2544 = £12.544$$

The future value of £1

	Interest rate (percent per annum)				
Year	*1*	*2*	*5*	*12*	*15*
1	1.0100	1.0200	1.0500	1.1200	1.1500
2	1.0201	1.0404	1.1025	1.2544	1.3225
3	1.0303	1.0612	1.1576	1.4049	1.5209
4	1.0406	1.0824	1.2155	1.5735	1.7490
5	1.0510	1.1041	1.2763	1.7623	2.0113

Over five years the result is:

$$F = P(1 + i)^n$$

$$17.62 = 10(1 + 0.12)^5$$

The interest on the accumulated interest is the difference between the total arising from simple interest and that from compound interest:

$$£17.62 - £16.00 = £1.62$$

Almost all investments pay compound interest and so we will be using this throughout the book.

Present values

There are many occasions in financial management when you are given the future sums and need to find out what they are worth in present value terms today. For example, you wish to know how much you would have to put aside today which will accumulate, with compounded interest, to a defined sum in the future; or you are given the choice between receiving £200 in five years or £100 now and wish to know which is the better option, given anticipated interest rates; or a project gives a return of £1m in three years for an outlay of £800,000 now and you need to establish if this is the best use of the £800,000. By the process of discounting a sum of money to be received in the future is given a monetary value today.

Example 3

If we anticipate the receipt of £17.62 in five years' time we can determine its present value. Rearrangement of the compound formula, and assuming a discount rate of 12 percent, gives:

$$P = \frac{F}{(1 + i)^n} \text{ or } P = F \times \frac{1}{(1 + i)^n}$$

$$10 = \frac{17.62}{(1 + 0.12)^5}$$

Alternatively, discount factors may be used, as shown in Table 2.10. (This is an extract from Appendix II at the end of the book.) The factor needed to discount £1 receivable in five years when the discount rate is 12 percent is 0.5674, so the present value of £17.62 is:

$$0.5674 \times £17.62 = £10$$

TABLE 2.10
The present value of £1

	Interest rate (percent per annum)				
Year	*1*	*5*	*10*	*12*	*15*
1	0.9901	0.9524	0.9091	0.8929	0.8696
2	0.9803	0.9070	0.8264	0.7972	0.7561
3	0.9706	0.8638	0.7513	0.7118	0.6575
4	0.9610	0.8227	0.6830	0.6355	0.5718
5	0.9515	0.7835	0.6209	0.5674	0.4972

Examining the present value table in Appendix II you can see that as the discount rate increases the present value goes down. Also the further into the future the money is to be received, the less valuable it is in today's terms. Distant cash flows discounted at a high rate have a small present value; for instance, £1,000 receivable in 20 years when the discount rate is 17 percent has a present value of £43.30. Viewed from another angle, if you invested £43.30 for 20 years it would accumulate to £1,000 if interest compounds at 17 percent.

Determining the rate of interest

Sometimes you want to calculate the rate of return that a project is earning. For instance, a savings company may offer to pay you £10,000 in five years if you deposit £8,000 now, when interest rates on accounts elsewhere are offering 6 percent per annum. To make a comparison you need to know the annual rate being offered by the savings company. We need to find i in the discounting equation.

To be able to calculate i it is necessary to rearrange the compounding formula. Since:

$$F = P(1 + i)^n$$

First, divide both sides by P:

$$F/P = (1 + i)^n$$

(The Ps on the right side cancel out.)

Second, take the root to the power n of both sides and subtract 1 from each side:

$$i = \sqrt[n]{[F/P]} - 1 \text{ or } i = [F/P]^{1/n} - 1$$

$$i = \sqrt[5]{£10{,}000 / £8{,}000} - 1 = 0.046 \text{ or } 4.6\%.$$

Not a good deal compared with other accounts offering 6%.

Example 4

In the case of a five-year investment requiring an outlay of £10 and having a future value of £17.62 the rate of return is:

$$i = \sqrt[5]{\frac{17.62}{10}} - 1 = 12\%$$

$$i = [17.62 / 10]^{\frac{1}{5}} - 1 = 12\%$$

Technical aside

You can use the $\sqrt[x]{y}$ or the $\sqrt[y]{x}$ button, depending on the calculator.

Alternatively, use the future value table, an extract of which is shown in Table 2.9. In our example, the return on £1 worth of investment over five years is:

$$\frac{17.62}{10} = 1.762$$

In the body of the future value table look at the Year 5 row for a future value of 1.762. Read off the interest rate of 12 percent.

An interesting application of this technique outside finance is to use it to put into perspective the pronouncements of politicians. For example in 1994, John Major made a speech to the Conservative Party conference promising to double national income (the total quantity of goods and services produced) within 25 years. This sounds impressive, but let us see how ambitious this is in terms of an annual percentage increase.

$$i = \sqrt[25]{\frac{F}{P}} - 1$$

F, future income, is double P, the present income.

$$i = \sqrt[25]{\frac{2}{1}} - 1 = 0.0281 \text{ or } 2.81\%$$

The result is not too bad compared with the previous 20 years. However, performance in the 1950s and 1960s was better and Asian country growth rates are generally between 5 and 10 percent.

The investment period

Rearranging the standard equation so that we can find n (the number of years of the investment), we create the following equation:

$$F = P(1 + i)^n$$

$$F/P = (1 + i)^n$$

$$\log(F/P) = \log(1 + i)n$$

$$n = \frac{\log(F/P)}{\log(1 + i)}$$

Example 5

How many years does it take for £10 to grow to £17.62 when the interest rate is 12 percent?

$$n = \frac{\log(17.62/10)}{\log(1 + 0.12)}$$

Therefore $n = 5$ years.

An application outside finance

How many years will it take for China to double its real national income if growth rates continue at 10 percent per annum?
Answer:

$$n = \frac{\log(2/1)}{\log(1 + 0.1)} = 7.3 \text{ years (quadrupling in less than 15 years)}$$

Consider the geopolitical implications of this!

Annuities

Quite often there is not just one payment at the end of a certain number of years. There can be a series of identical payments made over a period of years.

For instance:

- bonds usually pay a regular rate of interest;
- individuals can buy, from saving plan companies, the right to receive a number of identical payments over a number of years;
- a business might invest in a project which, it is estimated, will give regular cash inflows over a period of years;
- a typical house mortgage is an annuity.

An annuity is a series of payments or receipts of equal amounts. We are able to calculate the present value of this set of payments.

Example 6

For a regular payment of £10 per year for five years, when the interest rate is 12 percent, we can calculate the present value of the annuity by three methods.

Method 1

$$P_{an} = \frac{A}{(1 + i)} + \frac{A}{(1 + i)^2} + \frac{A}{(1 + i)^3} + \frac{A}{(1 + i)^4} + \frac{A}{(1+i)^5}$$

where A = the periodic receipt.

$$P_{10,5} = \frac{10}{(1.12)} + \frac{10}{(1.12)^2} + \frac{10}{(1.12)^3} + \frac{10}{(1.12)^4} + \frac{10}{(1.12)^5} = £36.05$$

Method 2
Using the derived formula:

$$P_{an} = \frac{1 - 1 / (1 + i)^n}{i} \times A$$

$$P_{10,5} = \frac{1 - 1 / (1 + 0.12)^5}{0.12} \times 10 = £36.05$$

Method 3
Use the 'Present Value of an Annuity' table. (*See* Table 2.11, an extract from the more complete annuity table in Appendix III.) Here we simply look along the year 5 row and 12 percent column to find the figure of 3.605. This refers to the present value of five annual receipts of £1. Therefore we multiply by £10:

$$3.605 \times £10 = £36.05$$

TABLE 2.11

The present value of an annuity of £1 per annum

Year	Interest rate (percent per annum)				
	1	*5*	*10*	*12*	*15*
1	0.9901	0.9524	0.9091	0.8929	0.8696
2	1.9704	1.8594	1.7355	1.6901	1.6257
3	2.9410	2.7232	2.4869	2.4018	2.2832
4	3.9020	3.5459	3.1699	3.0373	2.8550
5	4.8535	4.3295	3.7908	3.6048	3.3522

The student is strongly advised against using Method 1. This was presented for conceptual understanding only. For any but the simplest cases, this method can be very time consuming.

Perpetuities

Some contracts run indefinitely and there is no end to the payments. Perpetuities are rare in the private sector, but certain government securities do not have an end date; that is, the capital value of the bond will never be paid to the lender, only interest payments are made. For example, the UK government has issued Consolidated Stocks or War Loans that will never be redeemed. Also, in a number of project appraisals or share valuations it is useful to assume that regular annual payments go on forever. Perpetuities are annuities that continue indefinitely. The value of a perpetuity is simply the annual amount received divided by the interest rate when the latter is expressed as a decimal.

$$P = \frac{A}{i}$$

If £10 is to be received as an indefinite annual payment then the present value, at a discount rate of 12 percent, is:

$$P = \frac{10}{0.12} = £83.33$$

It is very important to note that to use this formula we are assuming that the first payment arises 365 days after the time at which we are standing (the present time or time zero).

Discounting semi-annually, monthly and daily

Sometimes financial transactions take place on the basis that interest will be calculated more frequently than once a year. For instance, if a bank account paid

12 percent nominal return per year, but credited 6 percent after half a year, in the second half of the year interest could be earned on the interest credited after the first six months. This will mean that the true annual rate of interest will be greater than 12 percent.

The greater the frequency with which interest is earned, the higher the future value of the deposit.

Example 7

If you put £10 in a bank account earning 12 percent per annum then your return after one year is:

$$10(1 + 0.12) = £11.20$$

If the interest is compounded semi-annually (at a nominal annual rate of 12 percent):

$$10(1 + [0.12/2])(1 + [0.12/2]) = 10(1 + [0.12/2])^2 = £11.236$$

The difference between annual compounding and semi-annual compounding is an extra 3.6p. After six months the bank credits the account with 60p in interest so that in the following six months the investor earns 6 percent on the £10.60.

If the interest is compounded quarterly:

$$10(1 + [0.12/4])^4 = £11.255$$

Daily compounding:

$$10(1 + [0.12/365])^{365} = £11.2747$$

Example 8

If £10 is deposited in a bank account that compounds interest quarterly and the nominal return per year is 12 percent, how much will be in the account after eight years?

$$10(1 + [0.12/4])^{4 \times 8} = £25.75$$

Converting monthly and daily rates to annual rates

Sometimes you are presented with a monthly or daily rate of interest and wish to know what that is equivalent to in terms of Annual Percentage Rates (APR) or Annual Equivalent Rate (AER).

If m is the monthly interest or discount rate, then over 12 months:

$$(1 + m)^{12} = 1 + i$$

where i is the annual compound rate.

$$i = (1 + m)^{12} - 1$$

If a credit card company charges 1.5 percent per month, the annual percentage rate (APR) is:

$$i = (1 + 0.015)^{12} - 1 = 19.56\%$$

If you want to find the monthly rate when you are given the APR:

$$m = (1 + i)^{1/12} - 1 \text{ or } m = \sqrt[12]{(1+i)} - 1$$

$$m = (1 + 0.1956)^{1/12} - 1 = 0.015 = 1.5\%$$

Daily rate:

$$(1 + d)^{365} = 1 + i$$

where d is the daily discount rate.